Having Fun With Non-Classical Logic

Howard A. Landman

or, Making Fun Of Classical Logic

Classical logic

- is the most widely used formal reasoning method
- has been in use for around 2000 years
- is the basis for most modern math and science
- is fully axiomatized and mathematically rigorous

But, it has some serious problems:

- fragile, can't handle contradictions or bad data

- doesn't model if-then statements in a way that matches natural language

A logic in which a single contradiction immediately causes all possible statements to be true is called *explosive*. Classical logic is explosive - *Ex Contradictione Quodlibet*. The following syllogism is true:

- John is 20 years old.
- John is 21 years old.
- Therefore, Mary is 40 feet tall.

"It is now standard to view [this] as a valid form of inference." - Stanford Encyclopedia Of Philosophy

However it violates our sense of *relevance*. What does John's age have to do with Mary's height?

A logic which is not explosive is called *para-consistent*.

Any logic which contains an explosive logic as a subset must also be explosive. Therefore, any paraconsistent logic must NOT contain classical logic as a subset, but must give at least one feature of classical logic up.

Different choices of which feature(s) lead to different logics.

Discursive logic models a conversation between multiple reasoning agents with different beliefs. A statement is true if any one of the agents can consistently believe it.

Example: One agent believes "Bush is evil"; another believes "Bush is good". Both are then considered true. However, "Bush is good AND Bush is evil" is not true because neither agent can consistently believe it.

Discursive logic gives up *conjunction*; just because "A" is true and "B" is true does not mean that "A AND B" is true.

Classical logic assumptions:

- All atomic statements have a truth value.

- All compound statements, built from operators and atomic statements, have a truth value.

- The only possible truth values are True and False.

What is the truth value of "if A then B"?

- False, if A is True and B is False.

- Otherwise True?

This means that "if A then B" is identical to "(not A) or B". This form of if is called *Material Implication* and written $A \supset B$.

The Paradoxes Of Material Implication

$(A \wedge B) \supset C \Rightarrow (A \supset C) \vee (B \supset C)$

If I close switch A and switch B, the light will go on.

Therefore, it is either true that if I close switch A , the light will go on, or that if I close switch B, the light will go on.

(What if the two switches are in series?)

The Paradoxes Of Material Implication

$$(L \supset E) \wedge (P \supset F) \Rightarrow (L \supset F) \vee (P \supset E)$$

If John is in London, then he is in England, and if John is in Paris, then he is in France.

Therefore, it is either true that if John is in London, then he is in France, or that if John is in Paris, then he is in England

The Paradoxes Of Material Implication

$$(L \supset E) \Rightarrow (\forall X, X \supset E) \vee (\forall Y, L \supset Y)$$

If John is in London, then he is in England.

Therefore, it is either true that if any statement X is True, then John is in England, or that if John is in London, then all statements Y are True.

(Consider $X =$ "John is not in England" and $Y =$ "John is not in London".)

The Paradoxes Of Material Implication

Classical Logic plus equating Material Implication to English if-then is an unsound reasoning method that gives erroneous conclusions!

Multi-Value Logic. One path to paraconsistent logic is to assume that there are more than 2 truth values. Consider the set:

- N = Neither true nor false
- T = True
- F = False
- B = Both true and false

We have to give up classical ideas:

- The Law of the Excluded Middle
  - There are no truth values besides T and F
- There are no true contradictions
  - T $\Rightarrow$ not F, F $\Rightarrow$ not T

Gödel Incompleteness Theorem showed that there is no complete, consistent model of arithmetic.

To be consistent, we must give up completeness. But what if we give up consistency?

In some paraconsistent logics, the Gödel proof fails. You *can not prove* that there must be true but unprovable statements. A complete (but inconsistent) theory of arithmetic seems possible.

"A foolish consistency is the hobgoblin of little minds ..."

- Ralph Waldo Emerson

Relevance Logic restricts if statements based on some idea of relevance.

"if A then B" is only allowed if A and B share some terms.

The relevance condition prevents ECQ.

Buddhist Logic Requires examples of both sides of every distinction. Arguments involving empty sets are not allowed.

- Where there is smoke, there is fire.
- Here there is smoke (like in a kitchen, unlike on a lake).
- Therefore, here there is fire.

Fuzzy Logic (Lotfi Zadeh)

Statements have a truth value from 0 to 1.

Operators also produce numbers, e.g. "$A \wedge B$" = MIN(A,B)

Similar to (but different from) reasoning about probability.

Conclusions

- Classical Logic, while powerful, is fragile with respect to errors and contradictions.

- Material Implication is **NOT** a good model of English if-then!

- CL + MI is an unsound reasoning method!

Conclusions

- Non-classical logics can be less fragile and (in some ways) more powerful.

- There is no general consensus on the "best" non-classical logic.

- Caveat Cogitator!

Further Reading

- Graham Priest, *Introduction to Non-Classical Logic*
- Graham Priest, *On Contradiction*