QUANTUM PHASE SHIFT CONSIDERED AS BEING DUE TO A GR-LIKE TIME DILATION

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Contents

1. Theory

1.1. **Introduction.** In general relativity, clocks at different heights in a gravitational field run at different rates, with the higher one running faster. In quantum mechanics, particles at different energy levels rotate their quantum phase at different rates, with the higher-energy one rotating faster. We propose here that these two phenomena can be viewed as essentially identical.

In GR the time dilation is normally given as a function of position within a gravitational field, while in QM the phase shift is normally given as a function of the energy of a wave function. To compare them, we need to put them into common terms. We choose to do this by considering position in terms of energy, and phase shift in terms of time.

1.2. **Gravitational Time Dilation.** In general relativity, position in a gravitational field affects the speed of clocks, with higher ones running faster.

It is sufficient for our present purposes to consider only the simplest case of a uniform (non-curved) gravitational field with strength g. The time dilation T_d

between two observers at different heights is given by

$$T_d \equiv \frac{t_f}{t_s} = 1 + \frac{gh}{c^2}$$

where h is the height difference and c is the speed of light. (For a derivation of this result see e.g. [1], section 48. NOTE: While this formula is the one most commonly given, it is not exactly correct, as it gives inconsistent results for three clocks all at different heights, and predicts reversal of time for $h < -c^2/g$ but not for $h > c^2/g$. It is only valid in the limit of small h ($|h| << c^2/g$; for a gravitational force of 1 G, $c^2/g \approx 10^{16}$ meters or about 1 light year), or small g (the "weak field limit", $g << c^2/h$). The correct formula must be something like $T_d = e^{gh/c^2}$, which has none of the above problems. Either formula can, however, lead us to the desired result.)

For a particle (or clock) of mass m, the energy difference between the positions of the two observers is $\Delta E = mgh$, the amount of work required to raise the mass from the lower observer to the upper, so we can rewrite the previous equation as

$$T_d = 1 + \frac{mgh}{mc^2} = 1 + \frac{\Delta E}{E} = \frac{E + \Delta E}{E} = \frac{E_f}{E_s}$$

where $E = mc^2$ is the energy equivalent of the mass m. Thus we have that

$$\frac{t_f}{t_s} = \frac{E_f}{E_s}$$

or, subtracting 1 from both sides and choosing one of the clocks arbitrarily as our reference, that

$$\frac{\Delta t}{t} = \frac{\Delta E}{E}$$

The time dilation, as a fraction of the total time elapsed, is exactly equal to the energy increase, as a fraction of the original energy. This is the relation between time dilation and energy in GR.

1.3. **Quantum Phase Shift.** Standing wave solutions to the Schrödinger equation with energy E oscillate as $e^{-iEt/\hbar}$. Absolute phase appears impossible to measure and may in fact have no physical meaning whatsoever; however, relative phase can be easily observed through various interference experiments. For identical particles, or different trajectories of the same particle, higher energy will cause the wave function to rotate phase more rapidly. Labeling the energy levels f and s as before and defining $\Delta E \equiv E_f - E_s$, we get a relative phase shift

$$\Delta\phi(t) = -i\Delta E t/\hbar$$

For any wave at a constant energy level, and thus constant phase velocity, a shift in phase can be produced by a shift in time. If we set the phase shift due to ΔE to be equal to the phase shift due to a time delay Δt,

$$-i(E + \Delta E)t/\hbar = -iE(t + \Delta t)/\hbar$$

we get that

$$\Delta E \cdot t = E \cdot \Delta t$$

or

$$\frac{\Delta t}{t} = \frac{\Delta E}{E}$$

which is the same equation arrived at in the previous section.

An alternate path to the same result starts with L. de Broglie's original equation relating frequency to energy (Eq. 1.1.5 in [4])

$$h\nu_0 = m_0 c^2$$

where $E_0 = m_0 c^2$ is the rest mass energy as above. Then the energy ratio equals the frequency ratio which, by definition, equals the time dilation:

$$\frac{E_0 + m_0 g h}{E_0} = \frac{E_f}{E_s} = \frac{\nu_f}{\nu_s} = \frac{t_f}{t_s}$$

In summary, it appears entirely reasonable to view the phase shift as being due solely to time dilation, with the local clocks on different particle paths running at different rates. Indeed, any other interpretation seems problematic.

1.4. **History.** As shown above, it is not even required to have the Schrödinger equation; de Broglie's work of 1925 is sufficient, so that the present results *could* have been derived as early as 1926. Why they weren't seems rather mysterious.

The idea of using quantum phase interference of entangled pairs to measure time dilation was proposed by Hwang et al. in 2002 [3], so they clearly understood that a time dilation would cause a phase shift; however, only gravitational time dilation was considered. The idea that there might be some kind of time dilation associated with e.g. an electric potential was raised in 2004 [2], but the discussion there was vague and inconclusive.

1.5. **Thought Experiments.** In terms of pure interference effects, quantum time dilation appears to be experimentally indistinguishable from the standard interpretation of a phase shift without time dilation. Fortunately, there are many other forms of measurement which are not handicapped in this way.

1.5.1. *Experiment 1.* Two scientists, Fred and Sally, work in a lab at the foot of a mountain. They synchronize a pair of atomic clocks, and divide between them a pair of identical particles whose quantum phase has also been synchronized. As a control experiment, they wait until Fred's clock (and Sally's) shows exactly 1 hour and interfere the two particles. In the absence of decoherence, the phases should still precisely match. That is, $t_{1f} = t_{1s} = 1$ hour, $\Delta t_1 \equiv t_{1f} - t_{1s} = 0$, and $\Delta\phi_1 = 0$.

1.5.2. *Experiment 2.* As a second test, they synchronize as before. Fred now rides a computer-controlled cable car up the mountain, carrying his clock and particle with him, and immediately descends the same way. When he reaches the bottom, he and Sally note a slight time difference $\Delta t_2 \equiv t_{2f} - t_{2s}$ between their clocks. They wait one additional hour (on both clocks and then interfere their particles as before. They now measure a phase shift $\Delta\phi_2$ associated with ascending and descending the mountain; some of this might come from acceleration effects, and some from Fred being higher than Sally.

1.5.3. *Experiment 3.* Finally, having controlled for and measured all these effects, they synchronize once more. Fred ascends the mountain in exactly the same way as before, spends one hour by his clock at the summit, then descends. They interfere their particles and compare clocks. According to GR, their clocks should now be off, with Fred's clock showing a later time (t_{3f}) than Sally's (t_{3s}) due to gravitational time dilation. After adjusting for Δt_2 by computing $t_f \equiv t_{3f} - t_{2f}$ and $t_s \equiv t_{3s} - t_{2s}$, they would see a time dilation of

$$T_d \equiv \frac{t_f}{t_s} = 1 + \frac{gh}{c^2}$$

as in section 1. They also measure a phase shift $\Delta\phi_3$. After subtracting the phase shift from experiment 2 ($\Delta\phi_2$) to correct for any effects from ascending and descending, they would have a residual phase shift $\Delta\phi \equiv \Delta\phi_3 - \Delta\phi_2$ due entirely to Fred having been higher in the gravitational field than Sally.

The question is how to interpret this shift. In Fred's frame of reference at rest at the top of the mountain, the rate of phase rotation of his particle should have been perfectly normal, and hence the total rotation proportional to t_f. In Sally's frame of reference at rest in the lab, Fred's particle is at a higher energy level and should be rotating phase at a higher rate with the relative phase shift proportional to $t_s \Delta E$. Both their computations predict exactly the same phase shift. If Sally tries to adjust both for the higher energy and make a time-dilation adjustment at the same time, she will get the wrong answer. Either one works, but they cannot be used together. This is because they are the same thing viewed in two different ways.

(Note: We accept the notion of simultaneity presented by Einstein in 1907 ([5], see also [6] and chapter 7 of [7]).)

2. Limitations

In special relativity, the time dilation for a particle or observer moving at velocity v is $T_d = \sqrt{1 - v^2/c^2}$. The higher the velocity, the higher the kinetic energy, but the *slower* the clock is perceived to run. Thus kinetic energy does not appear to have the same relationship to time that potential energy does; even the sign is reversed. A further difficulty, as noted by de Broglie [4], is that while all observers agree on gravitational time dilation, the time dilation due to motion is reciprocal (I think your clock is slower than mine at the same time you think my clock is slower than yours). Working through these issues is beyond the scope of the current paper.

3. Experiment

In this section, we assume the time-dilation interpretation is correct. Changes in the rate of time flow and changes in level of potential energy are therefore the same thing. The electrons in different orbitals in the same atom must be viewed as having different time dilations, even though they span the same space and have the same (non-moving) center of reference. Relativity shows that time is not a universal absolute, but rather depends on the position and velocity of the observer. To this, we must now also add the energy of the observer.

This is experimentally testable. Measurable predictions include

- A charged clock, placed inside a conductive cage, will run faster when the cage is given the same polarity charge, thereby raising the energy level of the clock, and slower when the cage is given the opposite charge. Radioactive ions or unstable particles will decay faster (or slower) under similar circumstances.
- Likewise, their decay rate will be different on different sides of the solenoid in a magnetic Ehrenberg-Siday-Aharonov-Bohm setup even though they never encounter any field. Note that, although fairly weak fluxes are used in typical ESAB experiments because only $3.9 \cdot 10^{-7}$ gauss-cm^2 is required to rotate the phase by 2π[8], much stronger fields could be used to test the time dilation effect. MRI machines with 10 tesla (= 10^5 gauss) fields over areas greater than 100 cm^2 have been demonstrated, so fluxes of 10^7 gausscm^2 are quite feasible.
- Alternately, geometric confinement could be used to raise the energy, along lines discussed in section 3 of [9]. This could be used on uncharged particles such as neutrons, while the above 2 approaches require charged particles.

There are many other possibilities, but these few should suffice to demonstrate that the time dilation view makes different predictions than the phase shift view and that the differences are accessible to experimental test.

3.1. **Muon-based Experiments.** The muon, with a half life of 2.2 μS, might be an attractive candidate for an experiment. While muons are difficult to produce by fission, fusion, or nuclear decay, beams of muons can be generated via pion decay, and have been used e.g. to test the standard model's prediction of their anomalous magnetic moment (see [10] and its references). A survey of methods of muon production can be found in [11]. Beams of muons produced by pion decay are inherently 100% polarized with spin opposite to the direction of emission ([11], p.16). Beam sources may be continuous or pulsed. Muon detectors adequate to measure time dilation effects can be simple and inexpensive enough to be used in an undergraduate physics lab[12].

Let's first analyze the situation for an electron. A phase shift of 2π happens when

$$\Delta E \cdot t = 2\pi\hbar = h = E \cdot \Delta t$$

so that for an electron

$$\Delta t = \frac{h}{E} = \frac{h}{m_e c^2} = \frac{6.626 \cdot 10^{-34}\text{m}^2\text{kg/S}}{(9.109 \cdot 10^{-31}\text{kg}) \cdot (2.998 \cdot 10^8\text{m/S})^2} = 8.093 \cdot 10^{-21}\text{S}$$

is the time difference between electron paths when the interference pattern has been shifted by one full fringe. This requires a total flux of $3.9 \cdot 10^{-7}$ gauss-cm^2 as noted above, but we should be able to use fluxes at least 10^{13}-10^{14} that large, leading to feasible Δts in the range of 10^{-7}-10^{-6} seconds. (Indeed the Brookhaven E821 experiment[10] applied a field of 1.45T over a ring with radius 7.11m; if the field was uniform, the total contained flux was therefore about $2.3 \cdot 10^{10}$ gauss-cm^2.)

For the moment I assume that this Δt is also achievable for a muon. (Worst case, if wrong, is that it is 207 times smaller, the ratio of the muon mass to the electron mass,)

A 200 kV muon beam should travel roughly as fast as a 1 kV electron beam, or about 2% of the speed of light or $6 \cdot 10^6$ m/S. If we split this beam and send it around a solenoid with a radius of about 100 cm (and hence cross-sectional area 314 cm^2) we should be able to have each path be no longer than, say, 600 cm. With a 10T = 10^6 gauss field strength the total flux would be $3.14 \cdot 10^8$ gauss-cm^2 and the predicted $\Delta t = 8.09 \cdot 10^{-21} \frac{3.14 \cdot 10^8}{3.9 \cdot 10^{-7}}\text{S} = 6.51 \cdot 10^{-6}\text{S}$. The flight time of the muon would then be about 10^{-7} S. With a half life of 2.2 μS, and ignoring relativistic corrections, we would expect in the standard interpretation that a fraction $2^{-t/2.2\mu\text{S}}$ of the muons on each path would remain undecayed

$$2^{-0.1/2.2} = 2^{-1/22} = 2^{-0.04545...} = 0.969$$

so that about 3.1% of the muons would decay on each path. However, the relatively very large time dilation predicted by the present theory would cause the fast-time path to experience a total time $(t + \Delta t)$ of $(0.1 + 6.51)\mu$S so that approximately 7/8 of the muons would have decayed.

There is obviously a problem with the slow-time path though; it cannot end up with negative experienced time-of-flight. This is due to using the linearized weak-field approximation. (Correct calculation TBD.)

Going back to our previous relationship

$$T_d = \frac{T_f}{T_s} = \frac{6.61}{0.1} = 66.1$$

we would get that the ratio of fast and slow times inferred from decay numbers should be 66.1.

Note that not is not necessary to actually interfere the two beams to measure this effect. One could, for example, simply have a single beam of muons rotating in a cyclotron ring. A large confined and shielded flux through the ring should have a large effect on the decay rate of the muons; reversing the flux or the direction of rotation should reverse the effect.

It would also be possible to simply fire a beam of muons through the center holes of one or more shielded toroidal inductors, as was done in the elegant Hitachi experiment to demonstrate the ESAB effect [13]. Larger inductors with larger fluxes would be necessary, but these are commercially available and inexpensive.

4. Summary

We propose an alternate interpretation of the well known quantum phase shift as instead a time-dilation effect. The mathematics of this is essentially identical to that of gravitational time dilation in general relativity, indicating perhaps a deep connection between QM and GR. This interpretation is shown to have measurable consequences, and experiments are proposed that could test its validity.

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